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Dynamical Casimir effect with Dirichlet and Neumann boundary conditions

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Abstract

We derive the radiation pressure force on a nonrelativistic moving plate in 1 + 1 dimensions. We assume that a massless scalar field satisfies either Dirichlet or Neumann boundary condition (BC) at the instantaneous position of the plate. We show that when the state of the field is invariant under time translations, the results derived for Dirichlet and Neumann BC are equal. We discuss the force for a thermal field state as an example for this case. On the other hand, a coherent state introduces a phase reference, and the two types of BC lead to different results.

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1. Introduction

An intriguing feature of the static Casimir effect is the dependence of the force on the type of boundary conditions imposed on the field. For two parallel infinitely permeable plates the force turns out to be attractive, and in fact identical to the original Casimir result for a pair of perfectly conducting plates. On the other hand, the Casimir force between a perfectly conducting plate and a permeable one is, surprisingly, repulsive [2]. Due to this peculiarity, permeable plates have been considered recently in the literature in the context of the Casimir effect as well as cavity QED [3].

An analogous situation takes place for a scalar field in 1 + 1 dimensions: taking Neumann conditions at the boundaries results in the same attractive force obtained with Dirichlet BC, whereas in the mixed case the force is repulsive [4]. For reviews on the Casimir effect see [5] and references therein.

In the case of moving boundaries [6], the force exerted by vacuum fluctuations usually contains a dissipative component [7]. The amount of mechanical energy dissipated is converted into pairs of real particles [8]. These radiation reaction forces on moving bodies may appear even in the case of only one moving wall [9–11]. Connections between the dynamical Casimir effect and several interesting phenomena such as the Unruh–Davies effect [12], black–hole physics [13], sonoluminescence [14], mass corrections [15] and quantum decoherence [16] have been discussed in the literature [17].

In this paper, we extend the discussion of the role played by different BC to the dynamical effect. We compute explicitly the radiation pressure force for both Dirichlet and Neumann BC, and discuss the class of field states for which the force is the same for these two types of BC. We consider a single moving boundary in the nonrelativistic approximation, and a massless scalar field in 1+1 dimensions in a general quantum state. This paper is organized as follows: section 2 presents the results for Dirichlet BC. In section 3, we consider Neumann BC and show that these two BC lead to the same force when the field state is invariant under time translations. As an example of such a state, we discuss the case of a thermal field in section 4. In section 5, we consider a coherent state, which is an example of a field state that does not satisfy this symmetry. Section 6 is devoted to the conclusions and final remarks.

2. Dirichlet BC

In this section, we assume that the field vanishes at the instantaneous position of the plate, when measured in the Lorentz frame S' which is co-moving at a given time: $\phi'(x',t')|_{\mathrm{plate}} = 0$, where the prime quantities refer to S'. In terms of the laboratory coordinates, this BC is equivalent to [6]

$$\phi(\delta q(t), t) = 0. \tag{1}$$

We solve equation (1) in the long wavelength approximation, and assume the effect of the motion to be a small perturbation. We follow Ford and Vilenkin [10], and define

$$\phi(x,t) = \phi_0(x,t) + \delta\phi(x,t) \tag{2}$$

where ϕ_0 corresponds to the solution with a static plate at the origin and $\delta\phi$ is a small perturbation which takes into account the effect of the motion of the plate. Hence the unperturbed field ϕ_0 satisfies the wave equation $\Box\phi_0(x,t) = 0$ and the BC $\phi_0(0,t) = 0$. Its normal mode expansion in the half-space $x > 0$ is[4]

$$\phi_0(x,t) = \int_0^\infty \mathrm{d}\omega \sqrt{\frac{\hbar}{\pi\omega}} \sin(\omega x)\left[a_\omega\, \mathrm{e}^{-\mathrm{i}\omega t} + a_\omega^\dagger\, \mathrm{e}^{\mathrm{i}\omega t}\right] \tag{3}$$

with

$$\left[a_\omega, a_{\omega'}^\dagger\right] = \delta(\omega - \omega'). \tag{4}$$

A similar decomposition is written in the half-space $x < 0$, with the bosonic operators a_ω and their Hermitian conjugates $a_\omega^\dagger$ replaced by independent operators b_ω and $b_\omega^\dagger$.

The field operator $\delta\phi$ also satisfies the wave equation $\Box\delta\phi = 0$ and is submitted to a BC which can be obtained directly from equations (1) and (2) by taking the Taylor expansion around $x = 0$ and neglecting terms of second order in $\delta q(t)$:

$$\delta\phi(0,t) = -\delta q(t)\partial_x\phi_0(0,t). \tag{5}$$

The net force on the moving plate may be written in terms of the suitable component of the energy–momentum tensor as follows:

$$F(t) = \langle T^{11}(\delta q^-(t), t) - T^{11}(\delta q^+(t), t)\rangle \tag{6}$$

with

$$T^{11}(x,t) = \tfrac{1}{2}[(\partial_x\phi)^2(x,t) + (\partial_t\phi)^2(x,t)] \tag{7}$$

[4] We are using $c = 1$ throughout the text.

and $\delta q^{\pm} = \delta q \pm \epsilon$ (with $\epsilon \to 0^{+}$). The average $\langle \cdots \rangle$ is taken over an arbitrary field state. For simplicity, however, we assume that the state in the half-space $x > 0$ is defined with respect to the operators a_ω and $a_\omega^\dagger$ exactly as the state for the half-space $x < 0$ is defined in terms of b_ω and $b_\omega^\dagger$. In this symmetrical case, the net force vanishes to zero order of δq (plate at rest at the origin). As an example, if we consider the field to be at thermal equilibrium at temperature T at the half-space $x > 0$, then our assumption implies that the field on the left-hand side of the plate is also at thermal equilibrium at the same temperature.

For Dirichlet BC, the term $(\partial_t \phi)^2$ in (7) does not contribute to the force, and we find

$$T_D^{11}(\delta q^{+}(t), t) = \tfrac{1}{2}[(\partial_x \phi_0)^2(0^{+}, t) + \{\partial_x \phi_0(0^{+}, t), \partial_x \delta\phi(0^{+}, t)\} + \mathcal{O}(\delta q^2)] \tag{8}$$

where $\{A, B\}$ represents the anti-commutator of two given operators A and B.

The Fourier representation allows for an interesting physical interpretation of the dynamical Casimir effect. Moreover, it provides a simple ultraviolet regularization of the force (alternatively, one may employ the point-splitting method in the time domain [10]). When taking the Fourier transform of (6), we replace T^{11} by the rhs of (8). As discussed above, the terms independent of δq from each side of the plate cancel in the symmetrical case. On the other hand, the linear terms in δq add to give

$$\mathcal{F}(\omega) = -\int \frac{\mathrm{d}\omega'}{2\pi} \langle \{\partial_x \Phi_0(0^{+}, \omega'), \partial_x \delta\Phi(0^{+}, \omega - \omega')\} \rangle. \tag{9}$$

The integration variable ω' in (9) is the frequency of the unperturbed field Φ_0; we avoid any change of variable of integration, so as to conserve its physical interpretation.

We now solve for the perturbed field $\delta\Phi$ in terms of Φ_0. We take the solution that propagates outwards from the plate: $\delta\Phi(x, \omega) = \mathrm{e}^{\mathrm{i}\omega|x|}\delta\Phi(0, \omega)$, and from the Fourier transform of the rhs of (5) we derive

$$\partial_x \delta\Phi(0^{+}, \omega) = -\mathrm{i}\omega \int \frac{\mathrm{d}\omega'}{2\pi} \delta Q[\omega - \omega'] \partial_x \Phi_0[0^{+}, \omega']. \tag{10}$$

Equation (10) shows that the scattering of a given field Fourier component ω' generates a new frequency (or sideband) at frequency $\omega' + \Omega$, where Ω is the mechanical frequency. We replace equation (10) into (9) to find

$$\mathcal{F}_D(\omega) = \mathrm{i} \int \frac{\mathrm{d}\omega'}{2\pi} (\omega - \omega') \int \frac{\mathrm{d}\omega''}{2\pi} \delta Q(\omega - \omega' - \omega'') \sigma_D(\omega', \omega'') \tag{11}$$

where we have defined the correlation function of the unperturbed field operator

$$\sigma_D(\omega, \omega') = \langle \{\partial_x \Phi_0(0^{+}, \omega), \partial_x \Phi_0(0^{+}, \omega')\} \rangle. \tag{12}$$

For future reference, we derive, by taking the Fourier transform of (3), the result

$$\partial_x \Phi_0(0^{+}, \omega) = \sqrt{4\pi\hbar|\omega|}\, \left[\theta(\omega) a_\omega + \theta(-\omega) a_{-\omega}^\dagger\right]. \tag{13}$$

From (13), we may derive the correlation function $\sigma_D(\omega, \omega')$ and hence the dynamical Casimir force for a variety of field states. However, before considering some specific examples, we consider in the next section the case of Neumann BC.

3. Neumann BC

In this section, we assume that the space derivative of the field, taken in the instantaneously co-moving Lorentz frame, vanishes at the plate's instantaneous position:

$$\partial_{x'} \phi'(x', t')|_{\text{plate}} = 0. \tag{14}$$

In the nonrelativistic approximation, and using the appropriate Lorentz transformation, this BC can be written in terms of quantities in the inertial frame of the laboratory as follows [11]:

$$\{\partial_x + \delta\dot{q}(t)\partial_t\}\phi(x,t)|_{x=\delta q(t)} = 0. \tag{15}$$

As in the previous section, we write $\phi = \phi_0 + \delta\phi$, but now the unperturbed field is given by

$$\phi_0(x,t) = \int_0^\infty \mathrm{d}\omega \sqrt{\frac{\hbar}{\pi\omega}} \cos(\omega x)\left[a_\omega\,\mathrm{e}^{-\mathrm{i}\omega t} + a_\omega^\dagger\,\mathrm{e}^{\mathrm{i}\omega t}\right] \tag{16}$$

whereas the motion induced correction satisfies, up to first order in $\delta q(t)$ and its time derivatives,

$$\partial_x\delta\phi(0,t) = -\delta q(t)\partial_x^2\phi_0(0,t) - \delta\dot{q}(t)\partial_t\phi_0(0,t) \tag{17}$$

which follows from the Taylor expansion of (15).

The force on the plate is computed from the energy–momentum tensor as discussed in the previous section. However, in contrast to the case of Dirichlet BC, here only the term $(\partial_t\phi)^2$ contributes in the expression of the rhs of equation (7), since (15) yields

$$(\partial_x\phi)^2(\delta q(t),t) = \delta\dot{q}(t)^2(\partial_t\phi)^2(\delta q(t),t) \sim \mathcal{O}(\delta\dot{q}(t)^2).$$

Hence, the net force on the moving plate can be written as

$$F_N = -\tfrac{1}{2}\langle(\partial_t\phi)^2(\delta q^+(t),t) - (\partial_t\phi)^2(\delta q^-(t),t)\rangle. \tag{18}$$

Keeping only the linear terms in δq, we obtain

$$F_N(t) = -\tfrac{1}{2}\langle\{\partial_t\phi_0(0^+,t), \partial_t\delta\phi(0^+,t)\} - \{0^+ \to 0^-\}\rangle + \mathcal{O}(\delta q^2). \tag{19}$$

As before, the contributions coming from terms involving only the field operator ϕ_0 cancel out, and the linear contributions from each side of the plate are equal, so that the net force on the plate can be written in the frequency domain as

$$\mathcal{F}_N(\omega) = \int \frac{\mathrm{d}\omega'}{2\pi}(\omega-\omega')\omega'\langle\{\Phi_0(0^+,\omega'), \delta\Phi(0^+,\omega-\omega')\}\rangle. \tag{20}$$

It is now convenient to express $\delta\Phi(0^+,\omega')$ in terms of $\partial_x\delta\Phi(0^+,\omega')$, since this last quantity can be written with the help of equation (17) (after a Fourier transformation) in terms of the unperturbed field Φ_0. The outward solution of the wave equation given the value of $\partial_x\delta\Phi(0,\omega)$ is

$$\delta\Phi(x,\omega) = \epsilon(x)\partial_x\delta\Phi(0,\omega)\frac{\mathrm{e}^{\mathrm{i}\omega|x|}}{\mathrm{i}\omega} \tag{21}$$

where ϵ denotes the sign function. Substituting equation (21) into equation (20), we obtain

$$\mathcal{F}_N(\omega) = -\mathrm{i}\int \frac{\mathrm{d}\omega'}{2\pi}\omega'\langle\{\Phi_0(0^+,\omega'), \partial_x\delta\Phi(0,\omega-\omega')\}\rangle. \tag{22}$$

As a last step, we take the Fourier transform of equation (17) and replace the result into (22):

$$\mathcal{F}_N(\omega) = -\mathrm{i}\int \frac{\mathrm{d}\omega'}{2\pi}\int \frac{\mathrm{d}\omega''}{2\pi}(\omega-\omega')\omega'\omega''\delta Q(\omega-\omega'-\omega'')\sigma_N(\omega',\omega'') \tag{23}$$

where we defined the correlation function of the unperturbed field operator

$$\sigma_N(\omega,\omega') = \langle\{\Phi_0(0^+,\omega), \Phi_0(0^+,\omega')\}\rangle \tag{24}$$

in analogy with (12). The unperturbed field appearing in (24) is computed from (16):

$$\Phi_0(0,\omega) = \sqrt{\frac{4\pi\hbar}{|\omega|}}\left[\theta(\omega)a_\omega + \theta(-\omega)a_{-\omega}^\dagger\right]. \tag{25}$$

By using equations (13) and (25), we derive the following relation between the correlation functions for Dirichlet and Neumann boundary conditions:

$$\sigma_D(\omega, \omega') = |\omega\omega'|\sigma_N(\omega, \omega'). \tag{26}$$

This result allows us to compare the forces for Dirichlet and Neumann boundary conditions. When the field state is invariant under time translations, the correlation function $\langle\phi_0(0, t)\phi_0(0, t')\rangle$ is a function of $t - t'$ only, and then in the frequency domain it satisfies $\sigma(\omega, \omega') \propto \delta(\omega + \omega')$. Hence we may replace $|\omega\omega'| = -\omega\omega'$ in (26), and from (11) and (23) it follows that Dirichlet and Neumann BC provide the same result:

$$\mathcal{F}_N(\omega) = \mathcal{F}_D(\omega) = \mathcal{F}(\omega). \tag{27}$$

Moreover, since $\omega' = -\omega''$ in (11) and (23), $\mathcal{F}(\omega)$ is proportional to $\delta Q(\omega)$, and may be written in terms of a susceptibility function $\chi(\omega)$ as follows:

$$\mathcal{F}(\omega) = \chi(\omega)\delta Q(\omega). \tag{28}$$

In the time domain, (28) reads

$$F(t) = \int \mathrm{d}t'\, \tilde{\chi}(t - t')\delta Q(t') \tag{29}$$

where $\tilde{\chi}(t)$ is the inverse Fourier transform of $\chi(\omega)$. Thus, the effect of the plate displacement at time t' on the force at time t depends only on $t - t'$, as expected from the assumption of time translational symmetry.

As an example of the field state obeying this assumption, we consider in the next section the force for a thermal field (temperature T). This example contains the case of the vacuum state as the limit $T = 0$. In section 5, we compute the force for a coherent state (amplitude α). This example also contains the vacuum state as a limiting case ($\alpha = 0$), but this time we find different results for the two types of BC when $|\alpha| > 0$.

4. Thermal state

We compute in the appendix the correlation function σ_D for a thermal field:

$$\sigma_D(\omega, \omega') = 4\pi\hbar|\omega|(1 + 2\overline{n}(\omega))\delta(\omega + \omega') \tag{30}$$

with

$$\overline{n}(\omega) = [\exp(\hbar|\omega|/k_B T) - 1]^{-1} \tag{31}$$

representing the average photon number at frequency ω (k_B is the Boltzmann constant). As expected and discussed in the previous section, $\sigma_D(\omega, \omega')$ is proportional to $\delta(\omega + \omega')$, a signature of time translational symmetry. Hence the force is the same for Dirichlet and Neumann BC, and it is given in terms of a susceptibility function $\chi(\omega)$ according to (28). From (11) and (30) we find

$$\chi(\omega) = 2\,\mathrm{i}\hbar \int \frac{\mathrm{d}\omega'}{2\pi}|\omega'|(\omega + \omega')\left[\frac{1}{2} + \overline{n}(\omega')\right]. \tag{32}$$

Thus the susceptibility is the sum of two contributions: $\chi = \chi_{\mathrm{vac}} + \chi_T$, where χ_T is proportional to $\overline{n}(|\omega'|)$ in (32), whereas χ_{vac} contains the effect of vacuum fluctuations (the term '1/2' in (32)). At the zero-temperature limit $\overline{n} = 0$ and then $\chi = \chi_{\mathrm{vac}}$.

Taking $\omega > 0$ (but an analogous argument is valid for $\omega < 0$) we can extract a finite result for $\chi_{\mathrm{vac}}(\omega)$ adopting the following regularization prescription:

$$\chi_{\mathrm{vac}}(\omega) = \mathrm{i}\hbar \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega'}{2\pi}|\omega'|(\omega + \omega') = \mathrm{i}\hbar \lim_{\Lambda\to\infty}\left(\int_{-\Lambda-\omega}^{0} + \int_{-\omega}^{0} + \int_{0}^{\Lambda}\right)\frac{\mathrm{d}\omega'}{2\pi}|\omega'|(\omega + \omega'). \tag{33}$$

Since the integrand is odd under reflection around $\omega' = -\omega/2$, the first and third integrals in the rhs of equation (33) cancel exactly for any value of Λ. Therefore, the single relevant contribution is the second term, which corresponds to negative frequencies that give rise to sidebands with positive frequencies (on the other hand, when considering a negative mechanical frequency ω, the sidebands are down shifted, and the contribution comes from the frequency interval $[0, -\omega]$). Positive and negative frequencies correspond to annihilation and creation operators, hence their mixture is clearly connected to the emission of photons out of the vacuum state. From equations (28) and (33), we find

$$\chi_{\mathrm{vac}}(\omega) = \mathrm{i}\hbar\omega^3/(6\pi) \tag{34}$$

or, in the time domain, $F_{\mathrm{vac}} = \frac{\hbar}{6\pi}\frac{\mathrm{d}^3}{\mathrm{d}t^3}\,\delta q(t)$, a result first obtained in [10], and which corresponds to the nonrelativistic limit of the exact expression of Fulling and Davies [9].

For the contribution of thermal photons, using that $\overline{n}(\omega') = \overline{n}(-\omega')$ we find from (32)

$$\chi_T(\omega) = 4\,\mathrm{i}\hbar\omega \int_0^\infty \frac{\mathrm{d}\omega'}{2\pi}\frac{\omega'}{\exp(\hbar\omega'/k_BT) - 1}. \tag{35}$$

The integral in (35) may be calculated in terms of the Riemann zeta function:

$$\chi_T(\omega) = \mathrm{i}\frac{2\pi(k_BT)^2}{3\hbar}\omega. \tag{36}$$

Adding up the results of (34) and (36) we find in the time domain

$$F(t) = \frac{\hbar}{6\pi}\frac{\mathrm{d}^3}{\mathrm{d}t^3}\delta q(t) - \frac{2\pi(k_BT)^2}{3\hbar}\frac{\mathrm{d}}{\mathrm{d}t}\delta q(t) \tag{37}$$

in agreement with [18]. By direct comparison of (34) and (36), the viscous, thermal contribution in (37) is much larger than the vacuum force when the typical Fourier components of $\delta q(t)$ are much smaller than $k_BT/\hbar$, which is of the order of $10^{13}\ \mathrm{s}^{-1}$ at room temperatures.

The thermal contribution does not result from the emission of new particles only. It is in part an effect of Doppler shifting the frequencies of the incoming thermal photons. The counterpropagating photons are up shifted when reflected by the plate by the amount $\delta\omega = 2\omega\delta\dot{q}(t)$, whereas the co-propagating photons are down shifted by the same amount. By momentum conservation, in both cases the plate recoils along the direction opposite to its motion. The resulting force is proportional to the total power P incident on both sides of the plate:

$$F_{\mathrm{Doppler}} = -2P\delta\dot{q}(t). \tag{38}$$

In the three-dimensional case, P is proportional to T^4 (Stefan–Boltzmann law) and so is the thermal viscous force [19], but in the one-dimensional case considered in this paper the thermal power is

$$P = \frac{\pi}{6}\frac{(k_BT)^2}{\hbar}$$

so that the viscous force as given by (37) is twice the Doppler force [18]. As discussed in the next section, a similar result holds for coherent states with Dirichlet BC.

5. Coherent state

The coherent state of amplitude α is defined as an eigenstate of the annihilation operator:

$$a_\omega|\alpha\rangle = \alpha\delta(\omega - \omega_0)|\alpha\rangle \tag{39}$$

where $\omega_0 > 0$ represents the frequency of the excited mode[5]. The symmetrical correlation function for the Dirichlet case is computed from (13) and (39):

$$\sigma_D(\omega, \omega') = \sigma_D^{\rm vac}(\omega, \omega') + \sigma_D^{\rm I}(\omega, \omega') + \sigma_D^{\rm II}(\omega, \omega') \tag{40}$$

with

$$\sigma_D^{\rm vac}(\omega, \omega') = 4\pi\hbar|\omega|\delta(\omega + \omega') \tag{41}$$

$$\sigma_D^{\rm I}(\omega, \omega') = 8\pi\hbar|\omega||\alpha|^2[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]\delta(\omega + \omega') \tag{42}$$

$$\sigma_D^{\rm II}(\omega, \omega') = 8\pi\hbar|\omega|[\alpha^2\delta(\omega - \omega_0) + \alpha^{*2}\delta(\omega + \omega_0)]\delta(\omega - \omega'). \tag{43}$$

$\sigma_D^{\rm vac}$ is the contribution of vacuum fluctuations, which are contained in the coherent state $|\alpha\rangle$. This term is also present in the thermal case, and corresponds to the zero-temperature limit ($\overline{n} = 0$) of (30). It originates from the commutator $\left[a_\omega, a_{\omega'}^\dagger\right]$ as given by (4). As expected, $\sigma_D = \sigma_D^{\rm vac}$ when $\alpha = 0$, since $|\alpha = 0\rangle$ is the vacuum state. The normally ordered correlation function is the sum of $\sigma_D^{\rm I}$ and $\sigma_D^{\rm II}$. The former originates from terms of the form $a_\omega^\dagger a_{\omega'}$, whereas the latter originates from terms $a_\omega a_{\omega'}$ and their Hermitian conjugates.

We calculate the force for Dirichlet BC from (11) and (41)–(43), and then for Neumann BC from (23) and (26). We write separately the contributions from each term in (40):

$$\mathcal{F}_{\substack{D\\N}}(\omega) = \chi_{\rm vac}(\omega)\delta Q(\omega) + \mathcal{F}^{\rm I}(\omega) \pm \mathcal{F}^{\rm II}(\omega) \tag{44}$$

with $\chi_{\rm vac}(\omega)$ given by (33) and

$$\mathcal{F}^{\rm I}(\omega) = \frac{4\,{\rm i}\hbar\omega_0}{\pi}|\alpha|^2\omega\delta Q(\omega) \tag{45}$$

$$\mathcal{F}^{\rm II}(\omega) = \frac{4\,{\rm i}\hbar\omega_0}{\pi}[\alpha^2(\omega - \omega_0)\delta Q(\omega - 2\omega_0) + \alpha^{*2}(\omega + \omega_0)\delta Q(\omega + 2\omega_0)]. \tag{46}$$

As expected, the contribution from $\sigma_D^{\rm vac}$ leads to the vacuum force already discussed in connection with the zero-temperature limit of the thermal field (see (34)). The contribution from $\sigma_D^{\rm I}$ is also of the form $\mathcal{F}^{\rm I}(\omega) = \chi^{\rm I}(\omega)\delta Q(\omega)$ and the same for the two types of BC, because $\sigma_D^{\rm I}(\omega, \omega') \propto \delta(\omega+\omega')$. However, the coherent state contains a phase reference, namely δ in $\alpha = |\alpha|\,{\rm e}^{{\rm i}\delta}$, which breaks the invariance under time translations. As a consequence, the forces for Dirichlet and Neumann BC are different due to the presence of the term $\mathcal{F}^{\rm II}$ in (44). As expected, the symmetry is restored if we average over δ, i.e. if we replace the coherent state by the incoherent statistical mixture defined by the density matrix operator

$$\rho = \int_0^{2\pi} \frac{{\rm d}\delta}{2\pi} ||\alpha|\,{\rm e}^{{\rm i}\delta}\rangle\langle|\alpha|\,{\rm e}^{{\rm i}\delta}|. \tag{47}$$

In fact, $\mathcal{F}^{\rm II}(\omega)$ vanishes if we take such an average, as can be checked in (46) or (perhaps more easily) from the expression in the time domain:

$$F^{\rm II}(t) = -\frac{4\hbar\omega_0}{\pi}|\alpha|^2[\cos(2\omega_0 t - 2\delta)\delta\dot{q}(t) + \omega_0\sin(2\omega_0 t - 2\delta)\delta q(t)]. \tag{48}$$

Since $\mathcal{F}^{\rm I}(\omega)$ does not depend on the phase δ, it does not change when the phase average is taken, and reads in the time domain

$$F^{\rm I}(t) = -\frac{4\hbar\omega_0}{\pi}|\alpha|^2\delta\dot{q}(t). \tag{49}$$

[5] The dynamics of a coherent state of the field inside a cavity with a moving mirror was considered in [20]. For a single moving mirror, the modification of the state of the field may be neglected when computing the force. Hence we take a prescribed field state.

It is useful to calculate the power P incident on the plate, in order to compare with the Doppler force as given by (38). With this end, we calculate the average of the energy–momentum tensor component $T_i^{01} = \{\partial_x\phi_i(x,t), \partial_t\phi_i(x,t)\}/2$ considering the incident field $\phi_i(x,t)$ alone, and taken in normal order so as to discard the contribution of vacuum fluctuations:

$$\left\langle : T_{i+}^{01} : \right\rangle = -\frac{\hbar\omega_0}{2\pi}[|\alpha|^2 + \mathrm{Re}(\alpha^2\,\mathrm{e}^{-2\,\mathrm{i}\omega_0(t+x)})] \tag{50}$$

for $x > 0$, and

$$\left\langle : T_{i-}^{01} : \right\rangle = \frac{\hbar\omega_0}{2\pi}[|\alpha|^2 + \mathrm{Re}(\alpha^2\,\mathrm{e}^{-2\,\mathrm{i}\omega_0(t-x)})] \tag{51}$$

for $x < 0$. The total incident power is

$$P = \left\langle : T_{i-}^{01} : \right\rangle(x = 0^-) - \left\langle : T_{i+}^{01} : \right\rangle(x = 0^+) = \frac{\hbar\omega_0}{\pi}|\alpha|^2[1 + \cos(2\omega_0 t - 2\delta)]. \tag{52}$$

We find the Doppler force for a coherent state by substituting (52) into (38). For Dirichlet BC, the sum of the viscous terms in (48) and (49) is twice the Doppler force, as in the case of the thermal field for both BC. This also holds for Neumann BC only if we average over the phase δ.

For a pure coherent state the force does not vanish when the plate is at rest, due to the second term in (48), which is proportional to the instantaneous position of the plate. This is a consequence of the spatial dependence of the energy–momentum tensor of the incident field (see (50) and (51)). On the other hand, for the thermal state and the incoherent mixture given by (47), translational symmetry forbids the existence of a net average force on a single static mirror.

6. Conclusion

Dirichlet and Neumann BC yield the same force on a moving mirror (in the nonrelativistic approximation) when the field state is symmetrical under time translations. This is the case for vacuum and thermal states, but not for a coherent state. The coherent state introduces a phase reference which breaks this symmetry. When averaging over the phase of the coherent state, we recover identical results for the two BC.

For both thermal and coherent states, the force may be split into two contributions: the first accounting for vacuum fluctuations, the second representing the contribution of the normally ordered correlation function. The latter vanishes in the limits of zero temperature (for the thermal state) and of zero amplitude (for the coherent state). Thus, the force for the vacuum state may be recovered from these two examples as a limiting case.

In both thermal and coherent cases the force contains a dissipative component proportional to the velocity of the mirror (viscous force). It is tempting to interpret this effect as a consequence of Doppler shifting the frequencies of the incident photons. However, the change of amplitude and phase by reflection is also determinant. In particular, the phase acquired by reflection underlies the difference between the results for Dirichlet and Neumann BC. No explicit connection between the viscous force and the Doppler effect holds for the coherent state with Neumann BC, whereas in the other examples discussed in this paper the viscous force was found to be twice the Doppler force.

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Appendix: Correlation function for a thermal state

In this appendix, we compute the correlation function $\sigma_D(\omega, \omega')$ for a thermal field, used in section 3 for deriving the force. From equation (13) we find

$$\sigma_D(\omega, \omega') = C(\omega, \omega') + C(\omega', \omega) \tag{A.1}$$

with

$$C(\omega, \omega') = 4\pi\hbar\sqrt{\omega|\omega'|}\theta(\omega)\theta(-\omega')\left\langle a_\omega a^\dagger_{-\omega'}\right\rangle + 4\pi\hbar\sqrt{|\omega|\omega'}\theta(-\omega)\theta(\omega')\left\langle a^\dagger_{-\omega} a_{\omega'}\right\rangle. \tag{A.2}$$

Using the thermal correlations

$$\left\langle a_\omega a^\dagger_{-\omega'}\right\rangle = (\overline{n}(\omega) + 1)\delta(\omega + \omega')$$
$$\left\langle a^\dagger_{-\omega} a_{\omega'}\right\rangle = \overline{n}(\omega')\delta(\omega + \omega')$$

where $\overline{n}(\omega)$ is the average thermal photon number defined in (31), we find

$$\sigma_D(\omega, \omega') = 4\pi\hbar[\omega\theta(\omega)(1 + 2\overline{n}(\omega)) + \omega'\theta(\omega')(1 + 2\overline{n}(\omega'))]\delta(\omega + \omega'). \tag{A.3}$$

Finally, we use that $n(-\omega) = n(\omega)$ and $\omega(\theta(\omega) - \theta(-\omega)) = |\omega|$ to derive (30) from (A.3).

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